

Algebraic identities and digit equations

Link squares through products and write two-digit numbers in place-value form

• Identities and two-digit place value

◆ Square expansions

- $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$.
- Rearrange to isolate sum of squares: $a^2 + b^2 = (a - b)^2 + 2ab = (a + b)^2 - 2ab$.
- $(a + b)^2 - (a - b)^2 = 4ab$ connects sum, difference and product directly.

○ Direct evaluation without roots

- Here the identity gives the required sum of squares without solving for a and b separately.
- Given $a - b = 4$ and $ab = 12$, substitute into $a^2 + b^2 = (a - b)^2 + 2ab$.
- Compute $4^2 + 2(12) = 16 + 24 = 40$; watch the $+2ab$ sign when starting from $(a - b)^2$.

◇ Place-value representation

- A two-digit number with tens digit a and units digit b equals $10a + b$, not ab .
- Reversing the digits swaps place values to give the new number $10b + a$.
- Digit bounds: tens digit $a \in \{1, \dots, 9\}$ and units digit $b \in \{0, \dots, 9\}$.

□ Reversal difference and solution

- Difference on reversal is always a multiple of 9: $(10b + a) - (10a + b) = 9(b - a)$.
- If reversal increases the number by 45, then $9(b - a) = 45$, so $b - a = 5$.
- Combining $a + b = 9$ with $b - a = 5$ gives $a = 2$, $b = 7$; original number is 27 ($72 - 27 = 45$).

REMEMBER Use $a^2 + b^2 = (a - b)^2 + 2ab$ for square sums, and $9(b - a)$ for two-digit reversal changes.