

Clock hands and relative angular speed

The 12-hour relative-angle cycle from coincidence to opposition and back

- 12-Hour Relative-Angle Cycle

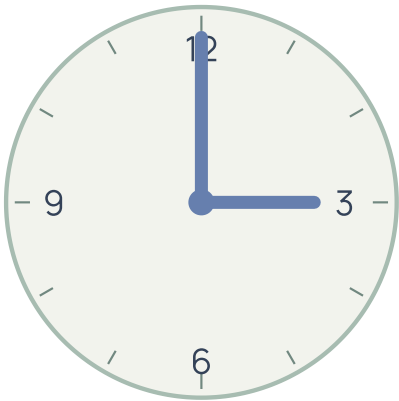


Illustration: 3:00 → 90°

Angular speed of clock hands

Minute hand: 6° per minute

Hour hand: 0.5° per minute

Relative speed = 5.5°/min

- Angular speeds

- At 12:00, the hands coincide; their relative separation is 0°.
- In 1 minute, the minute hand moves $360^\circ \div 60 = 6^\circ$ and the hour hand moves $30^\circ \div 60 = 0.5^\circ$.
- Thus the minute hand gains $6^\circ - 0.5^\circ = 5.5^\circ$ (or $11/2^\circ$) per minute over the hour hand.

- Angle calculation

- At h:m, with h in 0–11, raw separation is $d = |30h - 5.5m|$ degrees.
- Reduce d into 0°–360° and use $\min(d, 360^\circ - d)$ when the smaller angle between hands is asked.
- In each relative lap, hands reach 90° separation twice: at 90° and at 270° of relative gain.

- Opposite hands

- Hands form a straight line in opposite directions when the minute hand gains 180°.
- After 12:00, the first opposition occurs at $180 \div 5.5 = 360/11 = 32 \frac{8}{11}$ minutes past 12.
- Between 5:00 and 7:00, opposition occurs only once, exactly at 6:00 (not twice).

- Relative laps and counts

- A full relative lap of 360° takes $360 \div 5.5 = 720/11 = 65 \frac{5}{11}$ minutes to return to 0°.
- In 12 hours (720 min), relative gain is $720 \times 5.5^\circ = 3960^\circ = 11 \times 360^\circ$, or 11 full laps.
- Each relative lap has one opposition; count coincident endpoints only once. In 24 h: 22 oppositions.

REMEMBER Track the minute hand's 5.5°/min gain: 12 hours equal 11 relative laps, giving 11 oppositions in 12 hours and 22 in a day.