Convert recurring decimals to fractions

Align the repeating tail, subtract, reduce to lowest terms and check reciprocals

Recurring decimals and fraction form

01 Step 1: Identify the repeating block

- Distinguish a pure recurring decimal (0.232323…) from a mixed one (0.23333…).
- A reduced fraction p/q terminates only when q has no prime factors other than 2 and 5.
- Count non-repeating digits after the decimal point and digits in the repeating block.

↓

02 Step 2: Multiply to align tails

- For x = 0.232323…, two digits repeat, so multiply by $10^2 = 100$ to get 100x = 23.232323…
- Multiply y by 10 and 100 so both shifted forms, 10y = 2.333… and 100y = 23.333…, share the same tail.
- For y = 0.23333…, one non-repeating digit precedes one repeating digit.

↓

03 Step 3: Subtract and reduce

- Subtract aligned equations: 100x − x = 23 gives 99x = 23, so x = 23/99.
- Always reduce before summing terms: for p/q = 23/99 and r/s = 7/30, p + q + r + s = 159.
- For y: 100y − 10y = 21 gives 90y = 21, which reduces by gcd 3 to y = 7/30.

↓

04 Step 4: Check proper and reciprocal forms

- For positive integers a,b: a/b is proper if a < b and improper if a ≥ b.
- Reciprocal b/a is proper only when a > b; thus 9/7 has the proper reciprocal 7/9.
- Inverting a non-zero fraction swaps numerator and denominator: reciprocal of a/b is b/a.

REMEMBER Shift decimals until identical repeating tails cancel on subtraction, then reduce by the GCD.

Sources & practice · NCERT — Number systems · PET practice paper · 25-october-shift-2 · Q66 · PET practice paper · 30-october-shift-1 · Q43
mindmapsforupsc.com/mindmaps/pet-fractions-repeating-decimals/?lang=en