

Powers, roots and square arrangements

Use exponent laws, square checks and nearest perfect squares

- Powers, roots and square grids

- ◆ **Laws of exponents**

- For integer m,n and $a \neq 0$: $a^m \times a^n = a^{m+n}$ and $a^m \div a^n = a^{m-n}$.
- Power of a power or product: $(a^m)^n = a^{mn}$ and $(ab)^m = a^m b^m$.
- In $3^4 \times 3^x = 3^8$, matching exponents gives $4 + x = 8$, so $x = 4$.

- **Exact decimal square roots**

- When a terminating decimal is a perfect square, each pair of decimal digits gives one root digit.
- Always verify decimal placement by squaring: $1.5^2 = 2.25$, $0.08^2 = 0.0064$, $1.7^2 = 2.89$.
- Adding the verified roots: $1.5 + 0.08 + 1.7 + 1 = 4.28$.

- ◆ **Grouping cubes before dividing**

- Rewrite constants as powers before taking roots; for example, $8 = 2^3$.
- Combine bases under one exponent: $(9^3 \times 6^3) / 2^3 = ((9 \times 6) / 2)^3 = 27^3$.
- For $x^3 = 27^3$ with $x > 0$, the real cube root gives $x = 27$ directly.

- **Largest solid square grid**

- An $n \times n$ square grid uses n^2 items; the largest possible side is $\text{floor}(\sqrt{N})$.
- For 1000 items, $31^2 = 961$ and $32^2 = 1024 > 1000$, leaving $1000 - 961 = 39$ unused.
- For 1250 items, $35^2 = 1225$ and $36^2 = 1296 > 1250$, leaving $1250 - 1225 = 25$ unused.

REMEMBER Match bases before comparing exponents, verify decimal roots by squaring, and bracket N between consecutive squares.