

Count divisors using prime exponents

Every positive divisor is formed by choosing an exponent for each prime factor

Counting positive divisors of n

Prime factorisation first

- For a positive integer $n > 1$, write $n = p^a \times q^b \times r^c \dots$ using distinct primes.
- All bases p, q, r must be distinct primes; composite bases miscount divisors.
- Check completeness by multiplying the prime powers back to recover n .

Exponent-choice formula

- A prime power p^a offers $(a + 1)$ exponent choices: $0, 1, 2, \dots, a$.
- Choosing exponent 0 contributes $p^0 = 1$, which keeps 1 and other combinations included.
- Total positive divisors $d(n) = (a + 1)(b + 1)(c + 1)\dots$ by independent multiplication.

Worked check on 840

- Split step by step: $840 = 84 \times 10 = (4 \times 21) \times (2 \times 5) = 2^3 \times 3^1 \times 5^1 \times 7^1$.
- Add 1 to each prime exponent: $(3 + 1), (1 + 1), (1 + 1)$ and $(1 + 1)$.
- Multiply the counts: $4 \times 2 \times 2 \times 2 = 32$ positive divisors in total.

Factor pairs and squares

- Divisors pair up as $(d, n/d)$, so non-squares always have an even number of divisors.
- A number is a perfect square if and only if every prime exponent is even.
- When every exponent a, b, c is even, every factor $(a + 1)$ is odd, giving an odd divisor count.

REMEMBER Factorise into distinct primes, add 1 to each exponent for the zero-power choice, and multiply.